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Stochastic Model on finding the changes of temperature in Stratonovich Covariant differential Equation

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Abstract

In this paper, the changes of temperature in our delta districts in the specific days are considered and the result is to be compared with the stochastic model. In particular stratonovich covariant integral equation is applied in the mathematical model.

Key words: Stochastic differential equations in manifolds; Jump process; Stratonovich calculus; Covariant derivative; Tangent bundle.

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INTRODUCTION

Global temperature change, commonly referred to as 'global warming', is a very complex process in which global temperature trends have changed through time, especially in recent years, largely due to the impact of human industrialization and population growth (Karl Braganza *et al.*, 2004; Mraoua, 2007). Inthis study, we will focus on the temperature changes in the delta areas. Especially, on the first ten days of the months from June to December of the year 2018 are considered. We compare the departmental data with the mathematical model.

Our motivations come from both differential geometry and probability.We give reminders of the geometry and probability needed for placing our results in their context. We summarize Cohen's results about s.d.e with jumps in manifolds, and explain the particular case of Stratonovich equations. Next, we give definitions of a linear connection, yielding the notion of a covariant derivative and define continuous Stratonovich covariant s.d.e. on manifolds.

We expose the result about covariant stochastic differential equations (s.d.e) driven by semimartingales with jumps in a vector fibre bundle over a differentiable manifold (Norris, 1992). On the one hand, covariant differential equations are frequently used to intrinsically describe in a moving frame the evolution of some process.

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Reminder of Geometry and Probability

Stochastic Differential Equations with Jumps

2.1. Stochastic Differential Equations with Jumps

Definition:2.1.1

Let $\Psi:(A\times B)\times A\rightarrow B$ be a smooth map such that

1.$c\rightarrow\Psi((\mathrm{a,b}),\mathrm{c})$ is twice differentiable in a neighbourhood of the diagonal $\{((a,b),a),(a,b)\in A\times B\}$.

2.$\forall(a,b)\in A\times B:\Psi((\mathrm{a,b}),\mathrm{a})=\mathrm{b}$.

Then the map $a\rightarrow\Psi((\mathrm{a}_{t^-},\mathrm{b}_t),\mathrm{a})$ is called a point coefficient. It is a previsible and locally bounded process above (a_{t^-}).

Definition: 2.1.2

Let (a_t) be cadlag semimartingale on A, Ψ a point coefficient above (a_{t^-}), and b_0 an $\mathcal{F}_0$-measurable variable in B. We say that (b_t) is a solution of the s.d.e. with jumps

$$db=\Psi(b,d,a),\quad b_0=b_o,$$

If $(b_t)$is asemimartingale in B, such that $b_0=b_o$ and, for every θ in τ^* B, predictable and locally bounded above (b_{t^-}), we have

$$\int\theta db=\int\Psi(b)^*(\theta)da,b_0=b_o$$

where the process $(\Psi(b)^*(\theta)_t)$ is defined by

$$\forall a\in A,\quad \Psi(b)^*(\theta)_t(a)=\theta_t o\Psi((a_{t^-},b_{t^-}),a).(1)$$

Definition: 2.1.3

Let I be an interpolation rule on A. For every cadlagsemimartingale(a_t) on A, for z_0 in B, the stratonovich s.d.e. with jumps

$\partial^{\Delta} b_t = e(a_{t^-}, b_{t^-})\partial^{\Delta} a_t$, $b_0 = b_o$

Is defined as equivalent to

$\partial^{\Delta} b = \Psi(b, \partial^{\Delta} a)$, $b_0 = b_o$, (2)

with$\Psi((a_{t^-}, b_{t^-}), a_t) = \Psi(1)$, where $\Psi(s)$ is the solution of the differential equation
$\frac{\partial \Psi(s)}{\partial s} = e\left(I(s, a_{t^-}, a_t), \Psi(s)\right)\frac{\partial I(s, a_{t^-}, a_t)}{\partial s}, \Psi(0) = b_{t^-}$
(3)

Covariant Calculus

A connection on M is a smooth map $p: V \in TM \to pV$ such that, for every V=(x,c) in TM

1. pV projects$T_V TM$ onto the vertical space $\mathcal{V}_V TM = T_c(T_x M)$.
2. $\forall \lambda \in \mathbb{R}, p_{\overline{\lambda c}} o d\bar{\lambda}(V) = \lambda pc$ where $\bar{\lambda}(V) = (x, \lambda, c)$.

Every vector B=$dx^i(B)\frac{\partial}{\partial x^i} + dc^i(B)\frac{\partial}{\partial c^i}$ on $T_V TM$ can be uniquely written as $B = B^{\mathcal{V}} + B^{\mathcal{H}}$ with a vertical component $B^{\mathcal{V}}$ and a horizontal one $B^{\mathcal{H}}$, expressed as follows

$$B^{\mathcal{V}} = pc(B) = \left[dc^i(B) + \Gamma^i_{jk}(x)c^k dx^j(B)\right]\frac{\partial}{\partial c^i}, \quad (4)$$

$$B^{\mathcal{H}} = B - pc(B) = dx^i(B)\frac{\partial}{\partial x^i} - dx^k(B)\Gamma^i_{kj}(x)c^k\frac{\partial}{\partial c^i} . (5)$$

Seeing the tangent space $T_V TM$at $V = (x, c)$ as a product space $T_x M \times T_c(T_x M)$, we will sometimes write

$pc(B) = pc\left(dx^i(B)\frac{\partial}{\partial x^i}, dc^i(B)\frac{\partial}{\partial c^i}\right)$.

By the linearity of pc, we get

$$pc(B) = dx^i(B)pc\left(\frac{\partial}{\partial x^i}, 0\right) + dc^i(B)pc\left(0, \frac{\partial}{\partial c^i}\right)$$

Since pc is a projection on the vertical space, $pc\left(0, \frac{\partial}{\partial c^i}\right) = \frac{\partial}{\partial c^i}$. It follows that

$$pc(B) = dx^i(B)pc\left(\frac{\partial}{\partial x^i}, 0\right) + dc^i(B)\frac{\partial}{\partial c^i}$$

In particular, if the part $dx^i(B)\frac{\partial}{\partial x^i}$ of B on $T_x M$ and it is vertical part $pc(B)$ are known, we recover the part $dc^i(B)\frac{\partial}{\partial c^i}$ of B in $T_c(T_x M)$.

Covariant Derivative of a Curve

The covariant derivative of the curve $(Y_t) = ((x_t, y_t))$ is defined by

$\frac{\partial^{\mathcal{V}} Y_t}{\partial t} = pY_t\left(\frac{\partial Y_t}{\partial t}\right)$ (6)

In an imbedding, one has $\frac{\partial^{\mathcal{V}} Y_t}{\partial t} = \left[\frac{\partial y^i_t}{\partial t} + \Gamma^i_{jk}(x_t)y^k_t\frac{\partial x^j_t}{\partial t}\right]\frac{\partial}{\partial y^i}$,where

$\frac{\partial}{\partial y^i} \in T_{y_t}(T_{x_t} M)$. Notice our (non-usual) notation for the covariant derivative: the exponent $\mathcal{V}$ stands for "vertical", since the covariant derivative is valued in the interval space $T_{y_t}(T_{x_t} M)$.

In [1], Norris uses the following formula for the covariant derivative:

$\frac{\partial^{\mathcal{V}} Y_t}{\partial t} = \tau^{\parallel}_{0t}\frac{\partial(\tau^{\parallel}_{t0} Y_t)}{\partial t}$ (7)

Where $\tau^{\parallel}$ is the parallel transport along the projection curve (x_t) of (Y_t) on M. The equivalence between (6) and (7) is proved in (Maillard-Teyssier, 2003)

Continuous Stratonovich Covariant Calculus

Let p be a connection on M. For every continuous semimartingale$(Y_t) = ((x_t, y_t))$ on TM and every first order form α on TM, we define the Stratonovich covariant integral of α along (Y_t) by

$$\int \alpha Y_t\, \partial^{\mathcal{V}} Y_t = \int p^*(\alpha) Y_t\, \partial Y_t$$

In an imbedding, one has

$$\int \alpha Y_t\, \partial^{\mathcal{V}} Y_t = \int (\alpha Y_t)_i\left[\partial y^i_t + \Gamma^i_{jk}(x_t)y^k_t \partial x^i_t\right],$$

Where $(\alpha Y_t)_i = \alpha Y_t\left(\frac{\partial}{\partial y^i}\right), \frac{\partial}{\partial y^i} \in T_{y_t}(T_{x_t} M)$.

To study covariant stochastic differential equation, we work with two manifolds N and M, and consider semimartingale on TN and TM. From now on, e will be a smooth coefficient such that, for every W=(r,w) in TN and every Y=(x,y) is a linear map from$T_w(T_r N)$ to $T_y(T_x M)$. Notice that covariant s.d.e. are well defined only if the projection part (πY_t) on M of the solution (Y_t) in TM is known.

Lemma:2.4.1

For every coefficient e as above and connections $p^{(N)}$ on N and $p^{(M)}$ on M, define the map f on the manifold $(M \times TN) \times TM$, as follows.

For every $D = (x, A)$ in $M \times TN$ and every $Y = (x, y)$ in TM, $f(Z, Y)$ is given by $\forall(B_1, B_2) \in T_Z(M \times TN) \simeq T_xM \times T_ATN$,

$$f(Z,Y)(B_1,B_2) = e(A,Y)pA^{(N)}(B_2) - pY^{(M)}(B_1,0).$$

Then, f is a smooth map such that, for every $Z = (x, A)$ in $M \times TN$ and every $Y = (x, y)$ in TM, f(Z,Y) is linear from $T_Z(M \times TN)$ to $T_y(T_xM)$.

Proof

The smoothness of the map f comes from that of $(A, Y) \to e(A, Y)$ and that of $V \to pv$ for every connections p. Moreover, for every $Y = (x, y)$ in TM, the map $p_Y^{(M)}(.,0): T_xM \times \{0\} \subset T_YTM \to T_y(T_xM)$ (with $0 \in T_y(T_xM)$)

Is linear. For every Y=(x,y) in TM and A=(r,a) in TN, the maps $p_A^{(N)}: T_ATN \to T_aT_rN$ and $e(A, V): T_a(T_rN) \to T_y(T_xM)$ being linear, the map $e(A, V) \circ p_A^{(N)}: T_ATN \to T_y(T_xM)$ is linear. As a consequence, f is linear from $T_xM \times T_ATN$to $T_y(T_xM)$.

Proposition: 2.4.1

Let $p^{(N)}$ and $p^{(M)}$ be connections of order one, respectively on N and M. Let (A_t) be a continuous semimartingale in TN, (x_t) a continuous semimartingale in M, and Y_o an $\mathcal{F}_0$- measurable variable in TM.

We say that (Y_t) is a solution of the Stratonovich covariant s.d.e.

$$\partial^{\nu} Y_t = e(A_t, Y_t)\partial^{\nu} A_t Y_0 = Y_o \quad (8)$$

If (Y_t) is a continuous semimartingale in TM, such that $Y_0 = Y_o$and, for every form α on TM, we have

$$\int \alpha Y_t \, pY_t^{(M)} \partial Y_t = \int \alpha Y_t \, e(A_t, Y_t) pA_t^{(N)} \partial A_t \,, \quad Y_0 = Y_o$$

There exists a unique solution $(Y_t) = ((x_t, y_t))$ of the Stratonovich covariant s.d.e. (8), defined up to explosion.

Proof

Let us write (8) as $p_{Y_t}^{(N)}(\partial Y_t) = e(A_t, Y_t) p_{A_t}^{(M)}(\partial A_t)$. (9)

By remark 1, we have

$$p_{Y_t}^{(N)}(\partial Y_t) = p_{Y_t}^{(N)}(\partial x_t, 0) + \partial y_t.$$

Then (9) is equivalent to

$$\partial y_t = e(A_t, Y_t) p_{A_t}^{(M)}(\partial A_t) - p_{Y_t}^{(N)}(\partial x_t, 0). \quad (10)$$

Set $(Z_t) = ((x_t, A_t))$, semimartingale in $N \times TM$. Therefore, (9) is equivalent to the Stratonovichs.d.e.$\partial y_t = f(Z_t, Y_t)\partial Z_t$,Where f is the coefficient given bylemma 1.

This s.d.e. with linear coefficient f has a unique solution

$(Y_t) = ((x_t, y_t))$in TM, defined up to explosion, which is also the unique solution of (8). As a consequence, (8) has a unique solution $(Y_t) = ((x_t, y_t))$ in TM.

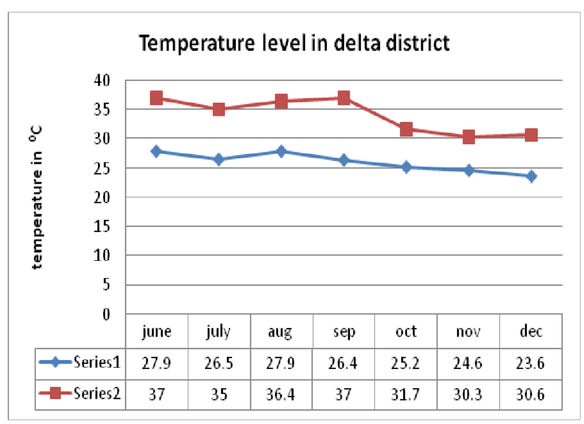
Case Study

The average of low and high temperature (in ºC) on the first ten days of the months from June to December of the year 2018in delta districts are given in the following Table 1.

Table.1. Temperature level in delta district

Month	Temperature level in delta district	
	Low Temperature	High temperature
June	27.9	37
July	26.5	35
August	27.9	36.4
September	26.4	37
October	25.2	31.7
November	24.6	30.3
December	23.6	30.6

The graphical representation of the temperature changes in delta districts for the considerable days with respect to the data in table 1 is shown in the figure 1.



Series-1: Low temperature

Series-2: High temperature

Fig.1. Temperature level in delta District

The mathematical modelon stratonovich covariant integral equation is applied to the given data in table 1 and the graphical representation is shown in figure 2.

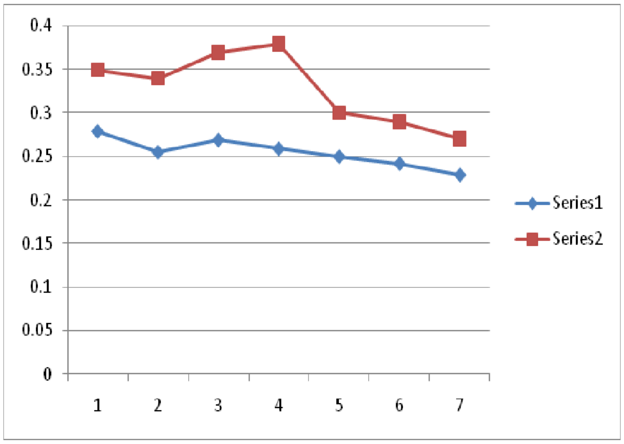


Fig.2.

CONCLUSION

The formalism developed in this paper concerns semi martingale in TM, but it may be generalized to processes in principal bundles over M. In this paper, we worked on a stochastic model on finding the temperature changes. In particular stratonovich covariant integral equation is applied for the mathematical report. Finally, on comparison of the temperature (low & high), the departmental report coincides with the mathematical report.

REFERENCES

Norris, J.R.1992. A complete differential formalism for stochastic calculus in manifolds, *Lecture Notes in Math.*, 1526 : 189-209. https://doi.org/10.1007/BFb0084322

Maillard-Teyssier, L.2005. Stochastic covariant calculus of order of two calculstochastique covariant d'ordnedeuxcomptesRendus, *Mathematique,* 341 : 185-188. https://doi.org/10.1016/j.crma.2005.06.023

Mraoua, M. 2007. Temperature stochastic modeling and weather derivatives pricing, African J. (online) *AfrikaStatistika*, 2 : 22-43.

Karl Braganza, David J. Karoly and Arblaster, J.M. 2004. Diurnal temperature range as an index of global climate change during the twentieth century, *Geographical Researach Letters*, 31. https://doi.org/10.1029/2004GL019998